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Measuring risk of crude oil at extreme quantiles*[1]

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Abstract

The purpose of this paper is to investigate the performance of VaR models at measuring risk for WTI oil one-month futures returns. Risk models, ranging from industry standards such as RiskMetrics and historical simulation to conditional extreme value model, are used to calculate commodity market risk at extreme quantiles: 0.95, 0.99, 0.995 and 0.999 for both long and short trading positions. Our results show that out of the tested fat tailed distributions, generalised Pareto distribution provides the best fit to both tails of oil returns although tails differ significantly, with the right tail having a higher tail index, indicative of more extreme events. The main conclusion is that, in the analysed period, only extreme value theory based models provide a reasonable degree of safety while widespread VaR models do not provide adequate risk coverage and their performance is especially weak for short position in oil.

Key words: *WTI oil, Value at Risk, VaR, extremes, extreme value theory*

JEL classification: *C14, C22, C46, G17, G32*

1. Introduction

The rise of US benchmark oil West Texas Intermediate (WTI) to \$147/barrel in early July 2008 and its collapse to under \$34/barrel five months later could be highlighted as the biggest story in the recent history of oil. Commodities, such as oil, exhibit certain risk characteristics that are different from traditional financial assets like stocks and bonds. Users of these commodities generally assign a value to holding a physical commodity, as the futures contracts cannot be consumed directly

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and may not allow delivery of the physical asset at the desired time or location. Furthermore, many commodities, such as energy, can usually be stored only at high costs. As a result, supply or demand changes are translated immediately into price changes, which lead to higher volatility of commodity investments compared to traditional assets. Furthermore, supply and demand shocks occur more frequently and on a larger scale. Cold winters or natural disasters can lead to an unexpected increases in demand or decrease in the supply of oil products and a subsequent sharp increase in prices. Political instability in oil-exporting countries accounts for the additional variation in oil prices. Some degree of price volatility is inevitable and must be accepted, since some external factors simply cannot be controlled. The idea that rising oil price volatility serves to stifle economic activity and reduce asset values has by now become widely accepted in the literature and seems virtually axiomatic. For example, Yang, Hwang and Huang (2002) state that higher oil prices yield subsequent recessions in oil consuming nations, as oil prices are negatively correlated with economic activities. Lee, Ni and Ratti (1995) showed that it is not sufficient any more to explore the issue of causality of real oil price to the economy through 1992 with the oil price level variable but that volatility has to be taken into account. Ferderer (1996) backs their results in concluding that it is the surprise of an oil shock that implies a greater impact on real economic growth. Low volatility in the oil markets before a major oil price increase leads to a higher impact of the oil price shock on economy than a highly volatile oil price environment. Both these studies conclude that the surprise of an oil shock is the main factor of change in the economy.

Producers and consumers need to recognize this fundamental volatility and work to encourage a flexible and efficient response. Over the last couple of years, oil price volatility has become the most significant issue facing the oil industry and its customers. Crude oil and petroleum products markets have all exhibited extreme price changes. Sudden changes in oil prices have contributed to a climate of uncertainty for energy companies and investors and a climate of distrust among consumers and regulators. Whatever the adverse effects of oil price volatility, it seems likely that oil prices will remain volatile in the foreseeable future. Unless significant amount of surplus capacity in crude oil production and refining emerges, markets will also remain sensitive to actual or feared disruptions in supply, which will most likely keep prices highly volatile in the short-term. Many factors have been put forward to explain these extreme movements in oil prices, such as political decisions, OPEC quotas, weather conditions, armed conflicts, speculation, structural changes in demand for diesel and gasoline and many other factors (see International Energy Agency, 2001). For OPEC to receive a part of the blame for the current episode of volatility is quite understandable given OPEC's importance and its central position in the oil market. But explaining volatility in terms of OPEC's "price fixing" is not warranted. OPEC abandoned fixing the reference price in 1987, favouring a system in which OPEC sets production quotas based on its assessment of the market's call on OPEC supply. Oil prices fluctuate depending in part on how well OPEC does this calculus. Through the process of adjusting its production quotas OPEC can only hope to influence price movements. This adjustment process can prove quite problematic, at times inducing undesired price volatility. However, a number of other factors such as the decline in spare crude oil production capacity in the oil producing countries, and lower level of desired inventories and rapid decline in surplus capacity in the global refining industry also enhanced this transformation to a period of more volatility. The resulting tightly balanced market has become more sensitive to actual or threatened supply disruptions, and swings in demand are increasingly met by price changes rather than delivery from storage. Liberalization of trading markets and development of transaction tools such as derivatives and information technology seem only to further intensify volatility (Pindyck, 2004). Recently, a number of studies have been devoted to the influence of crude oil prices, since this source of energy is a key driver behind most economic activities. Higher oil prices have a direct impact on macroeconomic variables and events including inflation (Chen, 2009; Cologni and Manera, 2008), gross domestic product (Cologni and Manera, 2009; Gronwald, 2008; Narayan and Smyth, 2007), reduction of investment (Rafiq et.al., 2009; Hamilton, 2003) and recession (Jones et. al., 2004). Apart from the macroeconomic issues, the volatility in oil prices can leave oil market participants, both producers and consumers, with potentially heavy losses (see Cheong, 2009).

Because unexpected price changes are fundamentally determined by supply and demand imbalances, market participants in commodity markets strongly focus on economic models which relate supply and demand to "fundamental market variables". Moreover commodity markets are strongly shaped by storage limitations, convenience yield and seasonality effects. Next to the price changes which originate from fundamental supply and demand imbalances, price volatility can stem from the behaviour of some market participants who engage in (short-term) speculation. A clear example of this phenomenon was oil buying by investment banks, in mid 2008, which served to compensate for the losses connected to collateralized securities. Modelling of risk for commodity products thus represents an inherent complexity due to the strong interaction between the trading of the products and the supply and demand imbalances which stem from the state of the economy. Measurement of financial risks has greatly evolved in the last two decades, from simple indicator of market value, through more complex measures such as scenario analysis to modern stress testing, Value at Risk (VaR) and Expected shortfall (ES) measures. One of the most significant advances in the past two decades in the field of measuring and managing financial risks is the development and the ever-growing use of VaR methodology. VaR has become the standard measure that financial analysts use to quantify financial risks including the commodity risk. VaR is usually defined as a maximum potential loss in value of a portfolio of commodities with a given probability over a certain time horizon. The main advantage of VaR over other risk measures is that it is theoretically simple. VaR can be used to summarize the risk of individual positions in a commodity, such as oil, or a risk of large portfolio of assets. VaR reduces the risk associated with any portfolio of commodities or other assets to just one number, the expected loss associated with a given probability over a defined holding period.

In this paper we do not focus on long term forecasting of oil price risk but on the short term, day-to-day, purely econometric VaR model that accounts for the characteristics of WTI oil one-month futures return series. WTI (Western Texas Intermediate) oil is the benchmark for light sweet crude in the United States and is the highest priced crude. Owing to its low viscosity and negligible sulphur content, WTI crude is rated as high quality and primarily used in the production of gasoline. For the purpose of risk management over longer time periods an economic model, which takes account of supply and demand dynamics, would be more appropriate than a purely econometric time-series model. Majority of studies in the VaR literature deal with computation of VaR for traditional financial assets such as stocks, bonds and futures, and they usually focus on downside risk i.e. negative returns. In our research we focus on VaR figures for both long and short trading positions in oil. Long position risk is important for investors that bought into a commodity since the risk comes from a decrease in prices. Consequently, such an investor would be interested in the left tail of the return distribution. Short position risk is important for investors that short-sold a commodity since the risk comes from an increase in prices. Such an investor would be interested in the right tail of the return distribution.

Commodity price behaviour varies between commodities depending on the specific factors influencing the supply and demand of each commodity. However, several characteristics are common across most commodities:

1) Commodity prices tend to fluctuate in the short-term due to daily and seasonal variations in supply and demand, but revert toward a long-term equilibrium.

2) Commodity price volatility influences the level of commodity prices.

3) The long run equilibrium price can and does shift over time to reflect fundamental changes in the characteristics of supply and demand.

4) Price behaviour for different commodities varies, sometimes dramatically, based on the underlying characteristics of the supply and demand of the commodity.

Typically, prices of energy commodities have been more volatile than most other commodities. Demand for energy commodities tends to vary on a daily basis due to the direct impact of weather on demand, while there is generally a substantial lag between changes in prices and the corresponding changes in supply (Hirshleifer, 1988). In addition, energy industries tend to be very capital intensive, with high fixed costs, and relatively low variable costs of energy production resulting in relatively low, short-term, elasticity of supply. As a result, energy demand tends to vary substantially from season-to-season, while energy supply tends to be relatively stable. In this sense, energy tends to behave quite differently from most other commodities (see Energy Information Administration, 2002).

Measurement of risks associated with commodity markets is a relatively new field of research and a surprisingly small number of papers deal with this topic. Oil price risk management has not been extensively studied but oil volatility and dynamics have been studied to some extent, among others, in works of Birol (2001) and Henning, Sloan, De Leon (2003). The literature on measuring financial risks and volatility via VaR models in financial industry is vast, but Jorion (2001) and Dowd (2002) should be pointed out for their systematic and integrated approach to this subject. We could only find a couple of studies specifically analysing oil price risk measurement. Giot, Laurent (2003) investigate commodity futures including WTI returns in the period 1987 – 2002. They find that WTI returns are characterised by negative skewness and leptokurtosis and test the performance of skewed T ARCH, APARCH and RiskMetrics parametric models. In their study RiskMetrics performed rather poorly at confidence levels above 99%. Skewed T APARCH model performed well compared to other studied models, but still failed for WTI long positions at 99% VaR at 10% significance level and 99,5% VaR at 5% significance level. Žiković, Fatur (2007) investigate WTI oil returns over the period 2000 – 2006 and also find negative asymmetry and leptokurtosis. They find that parametric normally distributed VaR provides correct unconditional coverage at 90, 95 and 99% confidence levels both for long and short positions. These findings can probably be attributed to the fact that their out-of-sample period was relatively tranquil. Füss, Adams, Kaiser (2008) investigate Goldman Sachs long-only passive excess return indices for commodities, including energy index, in which WTI forms a major part. For the period 1991-2007 they also find negative asymmetry and leptokurtosis. They analyse normally distributed VCV model, Cornish-Fisher expansion of VCV, RiskMetrics, GARCH and CAViaR model. Their conclusion is similar to Žiković, Fatur (2007) that simpler VaR models can provide adequate risk coverage during the tranquil periods but during times of high volatility more sophisticated models such as GARCH and CAViaR should be used.

This paper has two basic goals. *Firstly,* we investigate what type of distribution best fits the extreme tails of WTI oil one-month futures returns. This information is equally important for risk management purposes as well as for pricing of structured commodity derivatives. *Secondly*, we test the performance of a wide array of VaR models in measuring the risk occurring in the far left and right tail of the return distribution of WTI oil one-month futures returns. To answer which VaR models adequately capture the market risk at high quantiles in WTI oil one-month futures returns, eleven VaR models are tested in the period from 2000 to 2009. VaR models are calculated for a one-day holding period and 95, 99, 99.5 and 99.9% risk coverage.

In the paper we test the following two assumptions:

H1: Both theoretically and empirically generalized Pareto distribution provides the best fit to the extreme tails of positive and negative WTI oil returns.

H2: Unlike the widespread models, at the extreme quantiles VaR and ES models based on the extreme value theory provide reliable risk forecasts.

The rest of the paper is organised as follows: in section 2 of the paper, a brief overview of empirical research into measurement of risk for crude oil is presented. Section 3 presents a theoretical background on extreme value theory and extreme value VaR estimation. Section 4 gives a description of the analysed data and methodology used. Investigation into the distributional properties of the left and right tail of the WTI returns is also presented in this section. Section 5 presents an analysis of VaR backtesting results for long and short trading positions. The conclusions are presented in the final section.

2. Extreme value theory framework

Presuming n observations of P&L time series, if X is independently and identically distributed (IID) drawn from some unknown distribution $F(x) = P(X \leq x)$, estimating extreme value (EV) VaR posses a significant problem because the distribution $F(x)$ is unknown. Help comes from Fisher-Tippett theorem (1928), which can be considered to have the same status in EVT as the central limit theorem has in the study of sums. The theorem describes the limiting behaviour of appropriately normalised sample maxima. We denote the maximum of the first n observations by $M_n = max(X_1, \ldots, X_n)$. Assuming that we can find sequences of real numbers $a_n > 0$ and b_n such that $(M_n - b_n)/a_n$ the sequence of normalized maxima, converges in distribution:

$$P\{(M_n - b_n)/a_n \leq x\} = F^n(a_n x + b_n) \xrightarrow[n\to\infty]{} H(x) \tag{1}$$

for some non-degenerate distribution function $H(x)$. If this condition holds we say that F is in the maximum domain of attraction of H: $F \in MDA(H)$. It was shown by Fisher & Tippett (1928) that:

$F \in MDA(H) \Rightarrow H$ is of the type H_x for some x.

Thus, if we know that suitably normalized maxima converge in distribution, then the limit distribution must be an extreme value distribution. It shows that as n gets large the distribution of tail of X converges to the generalized extreme value distribution (GEV):

$$H_{\mu,\sigma,\xi}(x) = \left| \begin{matrix} \exp\left(-\left[1+\xi(x-\mu)/\sigma\right]^{-1/\xi}\right) \\ \exp\left(-e^{-(x-\mu)/\sigma}\right) \end{matrix} \right. \quad if \quad \begin{matrix} \xi \neq 0 \\ \xi = 0 \end{matrix} \tag{2}$$

where x satisfies the condition $1 + \xi(x\text{-}\mu)/\sigma > 0$. GEV distribution has three parameters: location parameter (μ), which is a measure of central tendency, scale parameter (σ), which is a measure of dispersion and tail index (ξ), which is a measure of the shape of the tail. GEV distribution has three special cases:

- If $\xi > 0$, GEV distribution becomes a Fréchet distribution, meaning that $F(x)$ is leptokurtotic.
- If $\xi = 0$, GEV distribution becomes a Gumbel distribution, meaning that $F(x)$ has normal kurtosis.
- If $\xi < 0$, GEV distribution becomes a Weibull distribution, meaning that $F(x)$ is platykurtotic, which is usually not the case with financial data.

Mean and variance are related to location and scale parameters of GEV distribution as follows:

$$Mean = \mu + \left[\frac{\Gamma(1-\xi)-1}{\xi}\right]\sigma \xrightarrow[\xi \to 0]{} \mu + 0{,}577216\sigma \tag{3}$$

$$Variance = \left[\frac{\Gamma(1-2\xi)-\Gamma^2(1-\xi)}{\xi^2}\right]\sigma^2 \xrightarrow[\xi \to 0]{} \frac{\pi^2}{6}\sigma^2 \tag{4}$$

It is usual to obtain mean and variance from μ and σ, but one must be careful not to confuse the two since they differ significantly. This relationship truly holds only under the assumption that x_i tends to zero. In practice however, x_i is estimated and fixed so caution is advised when using this relationship. Quantiles of GEV distribution can be obtained by taking log of equation (2):

$$\log(H_{\mu,\sigma,\xi}(x)) = \begin{cases} -\left(1+\xi(x-\mu)/\sigma\right)^{-1/\xi} & \xi \neq 0 \\ -\exp(-(x-\mu)/\sigma) & \xi = 0 \end{cases} \quad if \tag{5}$$

Value of x is than calculated to get the quantiles or VaRs associated with a desired confidence level (cl). EV VaR is calculated as:

$$VaR_{cl} = \mu - \frac{\sigma}{\xi}\left[1-(-\log(cl))^{-\xi}\right] \quad \text{(Fréchet VaR, } \xi > 0\text{)} \tag{6}$$

$$VaR_{cl} = \mu - \sigma \log\left[\log(1/cl)\right] \quad \text{(Gumbel VaR, } \xi = 0\text{)} \tag{7}$$

The Fisher-Tippett theorem tells us that fitting of the GEV distribution should be done on data on sample maxima. Although this is not a problem when dealing with hydrology or meteorology it might present a serious problem when dealing with financial data. Using only sample maxima would lead to serious waste of information. Since there is only one maxima in any sample period we are disregarding all other extreme events and thus limiting our data set. For this reason the most widely accepted method of using EVT in finance is based on modelling the behaviour of extreme values above a high threshold. This method is usually named peaks over threshold approach (POT). POT approach extracts extremes from a sample by taking the exceedances over a predetermined threshold u. An exceedance of the threshold u occurs when a realization is higher than the threshold, $X_t > u$ for any t in $t = 1, 2,..., n$. An excess over u is defined by $y = X_i - u$. Provided a high threshold u, the probability distribution of excess values of X over threshold u can be defined as:

$$F_u(y) = P(X - u \leq y \mid X > u) \tag{8}$$

which represents the probability that the value of X exceeds the threshold u by at most an amount y given that X exceeds the threshold u. The excess distribution above the threshold u as the conditional probability can be defined as:

$$F_u(y) = \frac{P(X - u \leq y \mid X > u)}{P(X > u)} = \frac{F(y+u) - F(u)}{1 - F(u)}, \qquad y > 0 \tag{9}$$

Balkema, de Haan (1974) show that under MDA conditions given in equation (1) the generalized Pareto distribution (GPD) is the limiting distribution for the distribution of the excesses, as the threshold tends to the right endpoint. A positive measurable function *s(u)* can be found such that:

$$\lim_{u \uparrow \infty} \sup_{0 \leq x \leq \infty} \left| F_u(x) - G_{\xi, \sigma(u)}(x) \right| = 0 \quad \text{iff} \quad F \in MDA(H_\xi)$$

This theorem suggests that for sufficiently high threshold u, the distribution function of the excess observations may be approximated by the GPD. Since $x = y + u$ for all excedances, the following representation holds provided that $X>u$:

$$F(x) = [1 - F(u)] F_u(y) + F(u) \tag{10}$$

As the threshold u gets larger, the excess distribution $F_u(y)$ converges in limit to the GPD, which is defined as:

$$G_{\xi,\sigma,\mu}(x) = \begin{cases} 1 - \left(1 + \xi \dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} & if \quad \xi \neq 0 \\ 1 - e^{-(x-\mu)/\sigma} & if \quad \xi = 0 \end{cases} \tag{11}$$

$$x \in \begin{cases} [\mu, \infty] & if \quad \xi \geq 0 \\ [\mu, \mu - \sigma/\xi] & if \quad \xi < 0 \end{cases}$$

where ξ is the shape parameter, σ is the scale parameter, and μ is the location parameter. The relationship between the standard GDP $G_\xi(x)$ and GEV $H_\xi(x)$ is simple, such that:

$$G_\xi(x) = 1 + \log H_\xi(x) \quad if \quad \log H_\xi(x) > -1$$

When $\mu = 0$ and $\sigma = 1$, the representation is known as the standard GPD. The GPD embeds a number of other distributions. When ξ>0, F is in the Fréchet family and *Hx,b(u)* is ordinary Pareto distribution. This representation is the most relevant for financial time series analysis since they are usually characterized by heavy tails. For ξ>0, $E[X^k]$ is infinite for $k > 1/\xi$. The number of finite moments is ascertained by the value of ξ: if $0.25 \leq \xi \leq 0.5$ the second and higher moments are infinite; if $\xi \leq 0.25$, the fourth and higher moments are infinite, and so forth. When $\xi = 0$, the *F* is in the Gumbel family and *Hx,b(u)* is an exponential distribution and, if ξ<0, *F* is in the Weibull family and *Hx,b(u)* is a Pareto type II distribution.

In order to estimate the tails of the loss distribution, the result that, for a sufficiently high threshold *u*, *Fu(y) » Gx,b(u)(y)* is used. An approximation of *F(x)*, for *X*>*u*, can be obtained from equation (10):

$$F(x) = [1 - F(u)]G_{\xi,\sigma,u}(x-u) + F(u) \tag{12}$$

An estimate of *F(u)* can be obtained non-parametrically by means of the empirical cdf:

$$\hat{F}(u) = (n-k)/n \tag{13}$$

where *k* represents the number of exceedences over the threshold *u* and *n* number of observations. By substituting equation (12) into equation (13), the following estimate for *F(x)* is obtained:

$$\hat{F}(x) = 1 - \frac{k}{n}\left(1 + \hat{\xi}\frac{x-u}{\hat{\sigma}}\right)^{-\frac{1}{\xi}} \text{ provided that } G_{\xi,\sigma,u}(x) = 1 - \left(1 + \xi\frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}} \tag{14}$$

Where $\hat{\xi}$ and $\hat{\sigma}$ are the maximum likelihood estimators of ξ and σ. This equation can be inverted to obtain an unconditional quantile of the underlying distribution, which is actually VaR. For $cl \geq F(u)$ VaR is calculated as:

$$VaR_{cl} = q_{cl}(F) = u + \frac{\sigma}{\xi}\left(\left(\frac{1-cl}{\overline{F}(u)}\right)^{-\xi} - 1\right) = u + \frac{\sigma}{\xi}\left(\left(\frac{1-cl}{k/n}\right)^{-\xi} - 1\right) \tag{15}$$

To remedy the problems of unconditional estimation that is traditional in EVT McNeil and Frey (2000) developed a conditional quantile EVT approach under the assumption that the tail of the conditional distribution of the underlying GARCH process is approximated by a heavy-tailed distribution. They apply EVT to the conditional return distribution by using a two-stage method, which combines GARCH model with EVT in applying the residuals from the GARCH process. McNeil, Frey (2000) conditional extreme value (EVT-GARCH) VaR can be written as:

$$VaR_{cl,t} = \mu_t + \sigma_t VaR(Z)_{cl} \tag{16}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \qquad r_t = \mu_t + \sigma_t Z_t$$

$$Z = \left(\frac{x_{t-n+1} - \mu_{t-n+1}}{\sigma_{t-n+1}}, \ldots, \frac{x_t - \mu_t}{\sigma_t}\right)$$

$$VaR(Z)_{cl} = u_Z + \frac{\sigma_Z}{\xi_Z}\left(\left(\frac{1-cl}{\overline{F}(u_Z)}\right)^{-\xi_Z} - 1\right)$$

To estimate EVT risk measures it is necessary to estimate EVT parameters – μ, σ, and in the case of Fréchet distribution the tail index (ξ). Estimation of the tail index is the most problematic element of EVT estimation. Embrechts et al. (1997) suggests determining the tail index of the distribution via Hill estimator:

$$\hat{\xi}_{n,k}^{(H)} = k^{-1} \sum_{j=1}^{k} \ln X_{j,n} - \ln X_{k+1,n} \tag{17}$$

where k, the tail threshold (cut off) used in the Hill estimation has to be chosen arbitrarily, which is a major source of problems in practice. The Hill estimator is the average of the k most extreme observations, minus ($k+1$)th observation, which is next to the tail. There are two basic approaches to handling the trade off between bias and variance. The first approach, recommended by Embrechts et al. (1997), is based on estimating the Hill estimator for a range of k values and selecting the k values where the plot of the Hill estimator against k flattens out. Danielsson, de Vries (1997) suggest finding an optimal value of k that minimizes MSE loss function and, in regards to MSE, reflects an optimal trade off between bias and variance. Their procedure takes a second-order approximation to the tail of the distribution and uses the fact that k is optimal (in the MSE sense) at the point where bias and variance reduce at the same rate. We chose the value of threshold which minimizes Anderson-Darling statistic as proposed by Coronel-Brizio, Hernandez-Montoya (2005). The use of Anderson-Darling statistic is due to the fact that the corresponding weighting function puts more weight in the tails of the distribution. Under the assumption that a tail follows a Pareto law, the asymptotic distribution of Anderson-Darling statistic is known and we can use this distribution as a reference to determine an estimate of the cut off using a statistical approach.

3. Data analysis

WTI oil one-month futures daily returns are collected from Bloomberg web site for the period of nine years, 04.01.2000 - 05.01.2009, which includes the latest financial crisis and its' effects on global economy. To secure an adequate out-of-the-sample VaR backtesting period the out-of-the-sample data is formed by taking out 1,000 of the latest observations from the series. The rest of the observations (for the EV models the entire N – 1,000 observations and for most of the non EV models maximum of 500 observations) are used as a learning set needed for VaR starting values and volatility calibration. The calculated VaR figures are for a 1-day ahead horizon at 95, 99, 99.5 and 99.9 percent confidence levels. VaR models that are tested in this paper are: Normal simple moving average (VCV) VaR, RiskMetrics system, Historical simulation with rolling windows of 100, 250 and 500 days, BRW (time weighted) simulation with decay factors of 0.97 and 0.99, EGARCH-t parametric model, unconditional EVT approach using GPD, Barone-Adesi et al (1999) Filtered historical simulation (FHS) and McNeil, Frey (2000) conditional EVT approach.

Table 1 gives a summary of descriptive statistics and normality test for the entire sample and out-of-the-sample daily log returns.

Table 1: Summary descriptive statistics for WTI oil one-month futures returns in the period 04-01-2000 – 05-01-2009 and sub-period 14-01-2005 – 05-01-2009

Period	04.01.2000 - 05.01.2009	14.01.2005 - 05.01.2009
Descriptive statistics		
Mean	0,00027	0,00002
Median	0,00134	0,00114
Minimum	-0,16545	-0,12595
Maximum	0,16410	0,16410
St.Dev.	0,02537	0,02566
Skewness	-0,27269	0,20697
Kurtosis	7,01	8,13
Normality tests		
Shapiro Wilk/Francia	0,044	0,055
(p value)	0,00	0,00
Unit Root tests		
ADF (AR + drift)	-35,645	-24,301
P-P (AR + drift)	-48,886	-33,703

Source: Author's calculation

During the analysed period both the highest loss and the highest gain were recorded on Mondays. The highest decline in prices in the series (-16.55%) was recorded three days after September 11 2001 attack, and the highest increase in the price of oil (16.41%) was recorded on December 22 2008. Mean and median of daily returns significantly differ, which is in breach of normality assumption. Both mean and median differ from zero and show a significant positive trend. Skewness and excess kurtosis of the series are also significantly different from zero. Normality tests show that the daily WTI oil one-month futures returns are far from being normally distributed. In the sub-period of the latest 1,000 days up to the beginning of 2009 a difference from the entire period is visible in the value of skewness which switched from negative to positive. This significant change can be attributed to the run-up in energy prices during 2008. The theory suggests that commodities such as energy exhibit positive skewness. This is because, in the case of a fixed supply, a negative supply shock has a particularly strong impact on prices. Storable commodities can respond to negative supply shocks as long as supply is not depleted. The property of positive skewness that is often found in the literature usually refers to monthly returns, but cannot be considered a general property of commodity returns. Empirical findings from Giot, Laurent (2003), Žiković, Fatur (2007) and Füss, Adams, Kaiser (2008) contrasts sharply with this claim. Higher than normal values for kurtosis can be attributed to aggressive price swings in the price of oil. Upward oil price spikes are usually driven by either unexpected increases on the demand side, e.g., during unexpectedly cold winters or the demand side, e.g. closing of oil rigs due to natural disasters or armed conflicts. Conversely, downward spikes usually occur in the summer when storage is near full capacity, or in case of a global recession.

Ljung-Box, ACF, PACF and Engle's ARCH test show that there is significant autocorrelation and ARCH effects present in WTI oil one-month futures daily returns i.e. volatility tends to cluster together (periods of low volatility are followed by further periods of low volatility and vice versa), meaning that the WTI returns are not IID and ARMA-GARCH representation is necessary to capture the dynamics of the data generating processes of this time series. These findings are troubling for VaR models based on normality assumption, as well as for the nonparametric and semi-parametric approaches that are based on IID assumption, such as historical simulation and BRW approach. This is very indicative for risk managers, because elementary assumptions of many VaR models are not satisfied, meaning that VaR figures obtained for such models cannot be trusted. Table 2 shows the estimated GARCH coefficients for the entire and out-of-the-sample period, standard errors are given in parenthesis.

Table 2: ARMA-GARCH/EGARCH parameter estimates for WTI oil one-month futures returns

Period	Mean			Volatility				
	C	AR	MA	K	GARCH	ARCH	Leverage	d.f.
04.01.2000 - 05.01.2009		0.27919 (0.41)	-0.32199 (0.42)	9.23E-06 (4.2E-06)	0.94508 (0.0136)	0.039183 (0.0085)		T (8.72) (1.237)
13.01.2005 - 05.01.2009		-0.93198 (0.066)	0.90792 (0.075)		0.99266 (0.0065)	0.099341 (0.021)	-0.0449 (0.016)	T(20.67) (9.059)

T – Student's T distribution, ν – degrees of freedom

Source: Author's calculation

All coefficients, except for ARMA coefficients when observing at the entire 2000 – 2009 sample as a whole, are highly significant, so ARMA-GARCH effects in the return series exist in the period of our interest (2005 – 2009). When looking at the entire sample period ACF, PACF and Ljung-Box Q-statistic of standardised innovation detect no presence of autocorrelation in the standardised innovations from fitted ARMA(1,1)-GARCH(1,1)-t model, meaning that the conditional mean model (ARMA(1,1)) successfully captured the autocorrelation present in WTI one-month futures returns. Ljung-Box Q-statistic of squared standardised innovation and ARCH test detect no presence of heteroskedasticity, meaning that innovations are IID. Similar findings apply to out-of-the-sample period, which had to be modelled as an ARMA(1,1)-EGARCH(1,1)-t process to capture the leverage effect that was present in the data. Estimated degrees of freedom of the conditional T distribution indicate the presence of fat tails. This means the standardized residuals are not normally distributed even after taking GARCH effects into account.

To find which distribution provides the best fit to tails of WTI oil one-month futures returns we fit fat tailed, positively skewed distributions: lognormal, gamma, inverse Gaussian (IG) and generalized Pareto (GPD), along with exponential distribution as a benchmark to the empirical tails. As stated earlier EVT methods are applicable over a high threshold with the most problematic element being the choice of a suitable threshold. By setting the threshold too high we are left with only a few data points and increase parameter uncertainty. By setting the threshold too low we are losing the theoretical justification for the application of extreme value theory. We fit the selected fat tailed distributions to 2.5% left and right tail of the return distribution. Distributions are fitted using maximum likelihood estimation. The results of parameter estimation with standard errors given in parenthesis are presented in Table 3.

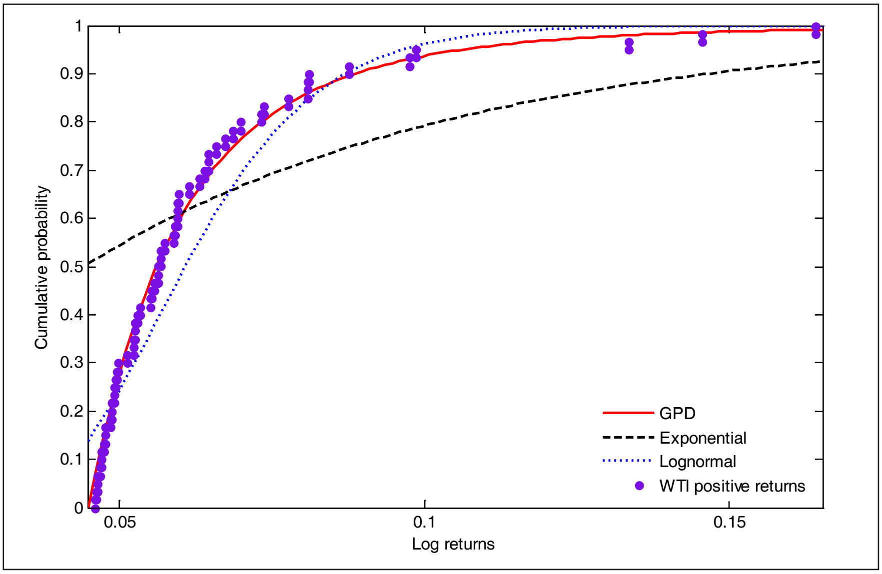
Table 3: Maximum likelihood parameter estimates and standard errors for the tested distributions

Negative returns					
Distribution	Lognormal	Exponential	Gamma	IG	GPD
Parameters	μ = -2.649	μ = 0.074	a = 11.164	μ = 0.074	ξ = 0.117
	(0.039)	(0.099)	(2.079)	(0.003)	(0.200)
	σ = 0.292		b = 0.0066	λ = 0.848	σ = 0.0203
	(0.028)		(0.001)	(0.160)	(0.005)
					k = 0.051
Log likelihood	138,36	89,83	135,63	138,47	155,51
Positive returns					
Distribution	Lognormal	Exponential	Gamma	IG	GPD
Parameters	μ = -2.801	μ = 0.0636	a = 11.110	μ = 0.064	ξ = 0.203
	(0.036)	(0.0082)	(1.999)	(0.002)	(0.149)
	σ = 0.281		b = 0.0057	λ = 0.773	σ = 0.0148
	(0.026)		(0.001)	(0.141)	(0.0029)
					k = 0.046
Log likelihood	159,61	105,31	154,25	159.23	180,42

Source: Author's calculation

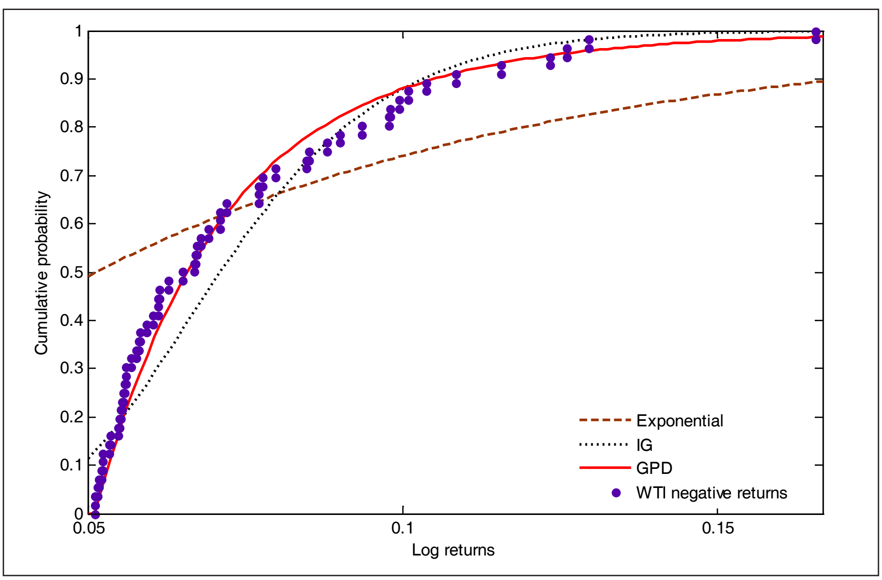
As the log likelihood shows, in both cases (left and right tail – long and short positions), GPD provides the best fit in the tails followed by the Inverse Gaussian and lognormal distribution. The exponential distribution does not fit the tail regions of the distribution well, clearly showing that both tails of the WTI oil one-month futures return distribution are fat tailed belonging to Fréchet domain of attraction. The two distributions providing the best fit to the empirical left and right tails, along with the worst fit – exponential distribution are plotted in figures 1 and 2.

Figure 1: Performance of GPD, lognormal and exponential distribution compared to empirical 2.5% right tail of WTI oil one-month futures returns



Source: Author's calculation

Figure 2: Performance of GPD, inverse Gaussian and exponential distribution compared to empirical 2.5% left tail of WTI oil one-month futures returns



Source: Author's calculation

Fitting the generalized Pareto distribution to WTI oil one-month futures tails in a 2.5 and 97.5% region is a useful method for estimating the behaviour of the tails of a distribution. The method has solid foundations in the mathematical theory of the behaviour of extremes and as such does not simply represent ad hoc curve fitting. It is possible that by trial and error, some other distribution can be found which fits the analysed tail data even better. An example of such a case can be found in Burneckia, Kukla, Weron (2000), where they find that for property claim services (PCS) indices lognormal distribution is superior to GPD in the tail region. One should keep in mind that such a distribution is an arbitrary choice, without any mathematical justification, and extrapolating beyond the available data set would be highly questionable.

4. Backtesting results

In this section the backtesting results for eleven VaR models are presented and their performance is analysed according to different criteria. Performance of each VaR model is evaluated separately for long and short position in the WTI oil one-month futures, based on several performance tests. Overall summary results are very useful to see how tested VaR model fare with standard backtesting framework based on the complete testing sample. Kupiec test and Christoffersen independence test are used to identifying VaR models that are acceptable to regulators, and actually provide the desired level of safety both to individual investors and regulators.

Kupiec and Christoffersen independence (IND) test backtesting results, at 5% significance level, for tested VaR models at 95, 99, 99.5 and 99.9% confidence levels are presented in tables 4 and 5.

Table 4: Kupiec test backtesting results at 95, 99, 99.5 and 99.9% confidence levels, period 14-01-2005 – 05-01-2009

VaR models	Positive returns				Negative returns			
	95%	99%	99,5%	99,9%	95%	99%	99,5%	99,9%
HS 100								
HS 250								
HS 500								
BRW λ=0,97	+							
BRW λ=0,99	+							
Normal VCV	+							
RiskMetrics	+					+	+	
GARCH	+				+	+	+	+
FHS	+	+			+	+	+	+
EVT GARCH		+	+	+	+	+	+	+
GPD	+	+	+	+	+	+	+	+

Grey areas mark the VaR models that satisfy Kupiec test for positive/negative WTI oil returns and selected confidence level, at 5% significance level.

Source: Author's calculation

Table 5: Christoffersen independence (IND) test backtesting results at 95, 99, 99.5 and 99.9% confidence levels, period 14-01-2005 – 05-01-2009

VaR models	Positive returns				Negative returns			
	95%	99%	99,5%	99,9%	95%	99%	99,5%	99,9%
HS 100	+	+	+	+	+	+	+	+
HS 250	+	+	+	+			+	
HS 500	+	+	+	+			+	+
BRW λ=0,97	+	+	+	+	+	+	+	+
BRW λ=0,99	+	+	+	+	+	+	+	
Normal VCV	+	+	+	+				+
RiskMetrics	+	+	+	+		+		+
GARCH	+	+	+	+	+	+	+	+
FHS	+	+	+	+	+	+	+	+
EVT GARCH	+	+	+	+	+	+	+	+
GPD	+	+	+	+		+	+	+

Grey areas mark the VaR models that satisfy Christoffersen IND test for positive/negative WTI oil returns and selected confidence level, at 5% significance level.

Source: Author's calculation

The backtesting results for the short position (positive returns) show that great majority of tested VaR models perform very poorly. None of the historical simulation models is capable of capturing the risk in the WTI oil one-month futures returns even at 95% confidence level. VCV, BRW simulation, RiskMetrics and EGARCH-t model are acceptable as a risk measure only at a 95% confidence level indicating that even more sophisticated parametric and nonparametric models did not yield any significant improvements over the simpler ones. FHS model is acceptable at 95 and 99% confidence levels but fails for higher quantiles. Extreme value models, unconditional GDP and conditional EVT-GARCH model provide superior risk coverage satisfying the backtesting criteria, with the only exception of conditional EVT-GARCH model failing at 95% confidence level. This finding can be viewed as a warning that extreme value models should only be used when measuring risk in the extreme tails of the distribution. All of the tested VaR models passed the Christoffersen independence test meaning that although they do not provide adequate risk coverage at least their VaR errors are IID i.e. they do not cluster together.

Backtesting results for long position (negative returns) differs to some extent from the results for short position. Historical simulation, VCV and BRW simulation models are not capable of capturing the downside risk in the WTI oil one-month futures returns even at 95% confidence level. RiskMetrics provides adequate risk coverage only at 99% confidence level. It passed the Kupiec backtesting criteria at 99 and 99.5% confidence levels but fails the independence test at 99.5%, indicating that it allows VaR errors to bunch together, which in practice is often even more dangerous than failing the Kupiec test. For the purpose of measuring downside risk the performance of EGARCH-t, FHS and conditional EVT models is superb, passing Kupiec and Christoffersen independence test for all confidence levels.

It is interesting to see that there is a significant difference in VaR model performance depending on the direction of the price change. This difference is especially visible for GARCH and FHS models which are well suited to downside risk measurement at high quantiles but are lacking in measurement of upside risk. Weak performance of widely used VaR models could be attributed to the fact that the time period under consideration includes the 2008 run-up in energy prices and the current slump caused by the ongoing global recession. Since we are using a sufficiently long out-of-the-sample backtesting period of 1,000 days (four years of daily data) these events should not be used as an excuse for the poor performance of VaR models. When taking into consideration the previously mentioned research into energy markets we can safely say that VaR models such as Historical simulation, VCV, BRW simulation and RiskMetrics are not suitable for measuring risk associated with oil prices at high quantiles. Out of the tested VaR models, EVT approach is shown to be the only acceptable approach to measuring risk at high quantiles for both long and short trading positions in oil.

Since VaR forecasts are mainly used to calculate capital reserves, and as such represent a cost, every institutional investor searches for a VaR model that neither under or overstates the true level of risk i.e. correctly reflects the true level of risk. By employing Lopez test and calculating average VaR value we identify which VaR model gives the closest fit to the true level of risk and as such is the most acceptable by investors. The results are presented in tables 6 and 7.

Table 6: Lopez test ranking of competing VaR models, period 14-01-2005 – 05-01-2009

VaR models	Positive returns				Negative returns			
	95%	99%	99,5%	99,9%	95%	99%	99,5%	99,9%
HS 100	14,99	16,37	10,26	13,21	21,77	14,22	11,14	11,11
HS 250	12,10	10,43	9,29	6,18	23,97	9,27	9,18	8,09
HS 500	10,15	11,47	7,32	4,15	18,14	15,39	7,23	4,09
BRW λ=0,97	4,83	15,40	15,35	13,32	9,59	12,22	13,20	12,16
BRW λ=0,99	7,91	9,36	8,31	10,29	12,71	6,19	7,16	5,15
Normal VCV	5,97	11,44	10,34	5,20	24,95	10,26	5,17	2,10
RiskMetrics	5,68	4,21	3,14	2,08	15,58	2,11	0,07**	2,03
GARCH	0,74	4,25	5,17	4,08	-4,58**	-0,95**	-0,98	-1**
FHS	-0,27**	2,22	4,14	3,05	-4,60	-6,98	-4,00	-1**
EVT GARCH	21,00	-2,84	0,07**	0,01**	7,48	-6,98	-4,00	-1**
GPD	-17,16	-0,85**	-6,95	0,01	-31,58	-4,95	-3,99	-1**

** marks VaR model with the lowest Lopez value i.e. smallest deviation from expected number of failures

Source: Author's calculation

Table 7: Average VaR values in percentage at 95, 99, 99.5 and 99.9% confidence levels, for VaR models which satisfied Kupiec test at 5% significance level, period 14-01-2005 – 05-01-2009

VaR models	Positive returns				Negative returns			
	95%	99%	99,5%	99,9%	95%	99%	99,5%	99,9%
HS 100								
HS 250								
HS 500								
BRW λ=0,97	3,65							
BRW λ=0,99	3,54							
Normal VCV	3,50**							
RiskMetrics	3,74					5,04**		
GARCH	3,81				3,77	5,33	5,91**	7,08**
FHS	3,84	5,50**			3,79	5,97	7,23	9,75
EVT GARCH		5,72	6,96**	10,37**	3,66**	5,69	6,58	8,69
GPD	4,20	9,82	13,15	15,66		9,86	11,80	16,85

** marks VaR model with the lowest average VaR value

Source: Author's calculation

For short trading position at 95% confidence level FHS model has the lowest Lopez size adjusted score, making it, by this criterion, the best VaR model since it minimises the deviation between recorded and expected VaR failure rate. For all other quantiles EVT models are the best performing models, deviating negligibly from the expected failure rates. In terms of the average VaR value, at 95 and 99% confidence levels, VCV and FHS models provides the lowest VaR value, minimising the cost of reserves. For higher quantiles conditional EVT-GARCH yields the lowest average VaR values. For long trading position, at 95 and 99% confidence level, EGARCH-t model has the lowest Lopez size adjusted score. At 99.5% RiskMetrics yields the lowest deviation from the expected value, but as we saw earlier it produces dependent VaR failures. At 99.9% confidence level there is a tie in the Lopez size-adjusted score between the EGARCH-t, FHS and EVT models. In terms of the average VaR value, at 95 and 99% confidence levels, EVT-GARCH and RiskMetrics models provided the lowest average value. At 99.5 and 99.9% confidence levels EGARCH-t yielded the smallest average VaR value.

5. Conclusion

We find that both theoretically and empirically generalised Pareto distribution fits the extreme tails of WTI oil one-month futures return distribution better than any other tested fat or medium tailed tested distribution. It could happen that by trial and error, some other distribution can be found which fits the analysed tail data even better. One should keep in mind that such a distribution is an arbitrary choice, without any mathematical justification, and extrapolating beyond the available data set would be highly questionable. This finding proves our assumption H1. From our results we can safely conclude that widely used VaR models such as Historical simulation, VCV, BRW simulation and RiskMetrics are not suitable for measuring risk associated with oil prices at high quantiles. Use of these models gives falsely optimistic information about the true levels of risk oil traders are exposed to. Out of the tested VaR models, EVT approach is shown to be the only acceptable approach to measuring risk at high quantiles for both long and short trading positions in oil, which proves our assumption H2. An obvious limitation of this study is that it focuses only on one brand of oil – WTI oil, while there are a dozen of equally important brands of oil. Our focus on just one brand and a very specific time period that is analysed leaves room for future research. An interesting characteristic to consider is the seasonality pattern in the oil returns. As it is clear, winter oil prices differ from the summer ones, so incorporating these facts into VaR models leaves significant room for future improvements of risk models in energy markets.

The fluctuation of oil prices is closely related to global macro events, financial markets movements and risk management. Understanding the stochastic process that lies beneath crude oil prices is important for policy makers, researchers and investors. The need for very accurate estimates of potential extreme changes in the price of oil be they positive or negative are equally important for investors, energy consumers/producers and states. Every time oil prices rise, economic activity in non oil producing countries declines by some measure. Any future cost stream that rises at a time when economic activity and asset values are in decline is highly risky. Besides standard risk measurement issues discussed in this paper our findings raise some interesting questions on the macroeconomic scale. Majority of the macro models use forecasts of volatility and risk inherent in oil price movements based on the standard models which rely on false assumptions. Our results show that standard models severely underpredict the true level of risk inherent in oil and that contemporary approaches to estimating extreme events should be used in order to avoid underestimating the effects of the main source of energy on economies.

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Mjerenje tržišnog rizika ulaganja u naftu pri ekstremnim kvantilima[1]

Saša Žiković[2]

Sažetak

Predmet ovog rada je istražiti uspješnost VaR modela u mjerenju tržišnog rizika jednomjesečnih futures ugovora na WTI naftu. Modeli mjerenja rizika, u rasponu od industrijskih standarda, kao što su RiskMetrics sustav i povijesna simulacija do kondicionalnog modela ekstremnih vrijednosti korišteni su u izračunu tržišnog rizika nafte pri ekstremnim kvantilima distribucije: 0,95, 0,99, 0,995 i 0,999 za duge i kratke trgovinske pozicije. Dobiveni rezultati pokazuju da od testiranih leptokurtičnih distribucija jedino generalizirana Pareto distribucija najbolje opisuje oba repa distribucije prinosa na naftu iako se oni sami međusobno značajno razlikuju, s time da desni rep distribucije ima znatno viši indeks repa što ukazuje na prisutnost ekstremnijih događaja. Naš glavni zaključak je da, u promatranom razdoblju, samo modeli temeljeni na teoriji ekstremnih vrijednosti uspješno predviđaju stvarnu razinu rizika pri kratkim i dugim trgovinskim pozicijama, dok rašireni modeli mjerenja pokazuju iznimno slabe rezultate, posebice kod mjerenja rizika kratkih trgovinskih pozicija.

Ključne riječi: *WTI nafta, rizična vrijednost, VaR, ekstremne vrijednosti, teorija ekstremnih vrijednosti*

JEL klasifikacija: *C14, C22, C46, G17, G32*

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